Sea State Analysis

Module 7 – CE A676 Coastal Engineering

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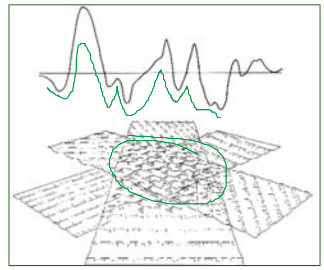
Topics

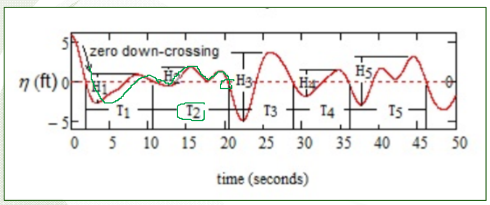
- Wave height distribution
- Wave energy spectra
- Wind wave generation
- Directional spectra
- Hindcasting
- Extreme sea states

Corresponding to Chapter 6 in text (Sorensen)

Wave height distribution

- Natural waves are rarely regular or sinusoidal in appearance
  - Many interacting waves give sea surface an irregular appearance
- Analysis of irregular $\eta(t)$:
  - Sum of 5 sinusoidal waves
  - Zero down-crossing id of a wave

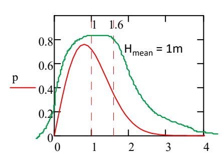


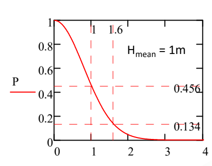


Rayleigh Distribution

- Seas generally assumed to have wave height Rayleigh distribution
  - Swell does not, but is often described by Rayleigh parameters
- For $x = \frac{H}{\overline{H}}$, wave heights relative to mean
  - probability density function: $$p(x) = \frac{\pi}{2} x e^{-\frac{\pi}{4}x^2}$$
  - probability distribution function,
    - probability that a particular wave height exceeds a specified height

$$P(x) = e^{-\frac{\pi}{4}x^2}$$

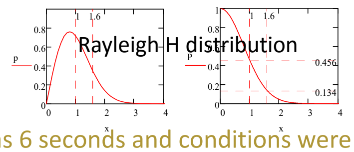




- $H_{1/10} = 1.27 H_{1/3} = 2.03\overline{H}$ $\quad H_{1/3} = 1.60\overline{H}$
- $H_{max} = 1.6 - 2.0 H_{1/3}$
- $H_{1/3} = \text{"significant wave height, } H_s\text{"}$

Wave period

- No such well-accepted distribution of wave periods
- Average period is commonly observed and reported
  - As with wave heights, visual observations miss shorter periods
  - Sometimes called "dominant wave period," which relates to energy spectrum characteristics



- *How many waves in a sea state?*
  - Suppose the average wave period was 6 seconds and conditions were stationary (statistics remained constant) for 1 hour ➡ 600 waves
  - Stationarity is often assumed to exist from one to 3 hours in wave monitoring strategies
    - *i.e.,* a measurement every 1 to 3 hours: order of 10^3 waves per sea state
  - Encountering a wave height 2 x H_s doesn't seem far-fetched!

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Wave energy spectrum

- Single-wave energy per unit surface area: $E = \frac{\rho g H^2}{8}$
  - Energy in physics generally proportional to amplitude squared
    - For example: $KE = \frac{1}{2} m v^2$
- Wave energy flux (Power): $P = \frac{nE}{T}$ $n = \frac{1}{2}\left(1 + \frac{2kd}{\sinh(2kd)}\right)$
  - A function of 1/T (frequency)
- Spectral analysis associates wave energy and frequency
  - Either $f = {}^1/_T$ (Hz, cycles per sec) or $\sigma = {}^{2\pi}/_T$ (radians per sec)
  - Sometimes period is used, usually as a secondary x-axis

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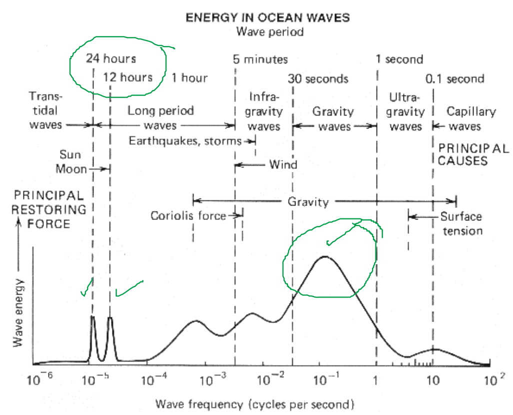
Energy in ocean waves

- Wind-induced free-surface gravity waves:
  - 1 < T < 30 seconds
  - 1 > f > 0.03 Hz
- Note narrow peaks at diurnal and semi-diurnal periods
- Engineering applications call for a spectrum of an individual sea state



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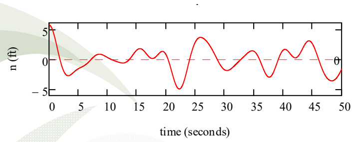
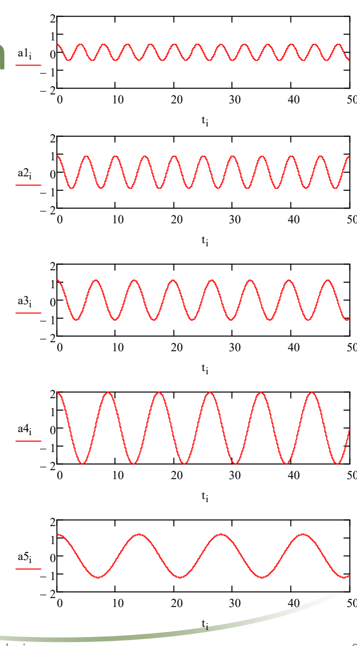
Fourier transformation

- Decompose a sea state into constituents
  - Each with an amplitude (H/2) and a frequency



- Associate constituent energy $\propto \left(\frac{H}{2}\right)^2$ with its corresponding frequency



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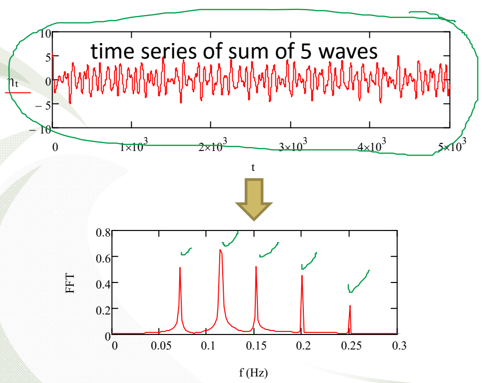
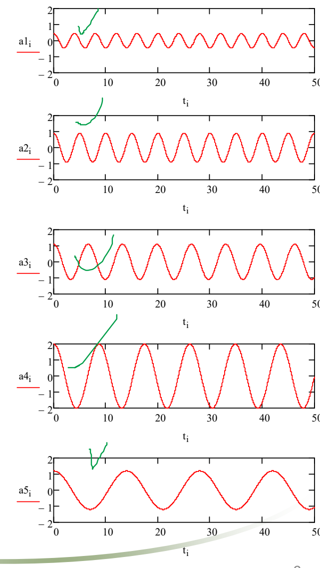
Fast Fourier Transformation

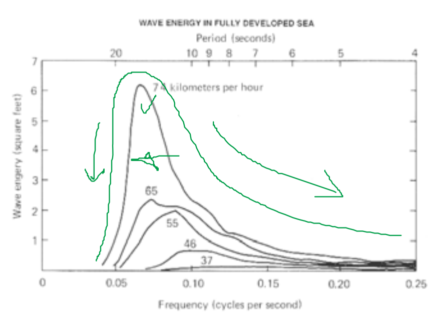
- FFT of 5-wave sum
  - shows the original frequencies, $f = {}^{1}/_{T}$
  - FFT y-value proportional to $({}^{H}/_{2})^2$





Sea State Spectra

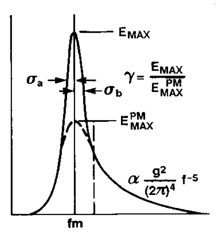
- Wind energy - wave energy correspondence
- Predict wave spectrum from wind conditions
  - Distribution is continuous
  - Steep on low frequency side
  - Tapered on high frequency side
  - Stronger winds
    - Higher peak energy
    - Lower peak frequency
      - Longer peak period



Empirical spectra

- Phillis (1958) $E(f) = \alpha g^2 f^{-5}$
  - *E(f)* is energy (m² or ft²) as a function of frequency, *f*
- Pierson-Moskowitz (1964) $E(f) = \alpha g^2 (2\pi)^{-4} f^{-5} e^{-\beta\left(\frac{g}{Uf}\right)^4}$
- JONSWAP (Joint North Sea Wave Project, 1973)

$$E(f) = \alpha g^2 (2\pi)^{-4} f^{-5} e^{-\frac{5}{4}\left(\frac{f_m}{f}\right)^4 + \ln \gamma e^{\left[-\frac{(f-f_m)^2}{2\sigma^2 f_m^2}\right]}}$$



  - α = equilibrium range constant or "Phillips" constant
  - f_m = *peak frequency*
  - γ = *peak enhancement factor*
    - increases peak energy above Pierson-Moskowitz
  - σ = *shape parameter*
    - $\sigma = \sigma_a$ for $f < f_m$
    - $\sigma = \sigma_b$ for $f > f_m$

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CEM[1] wind wave prediction

- JONSWAP-based: independent variables
  - Wind speed: an average value, generally over an hour
    - Not "gust speeds" for most coastal engineering purposes
    - Adjusted to equivalent value at 10 m above water
  - Duration: measured resolution rarely less than an hour
    - Values measured for shorter spans adjusted to 1-hr equivalent
  - Fetch: Distance over water across which the wind blows
    - Straight line in dominant direction of wind
    - Winds measured over land adjusted to over water equivalent
  - Depth: Depths other than deep water constrain wave growth

[1]Coastal Engineering Manual, Part II, Chapter 2

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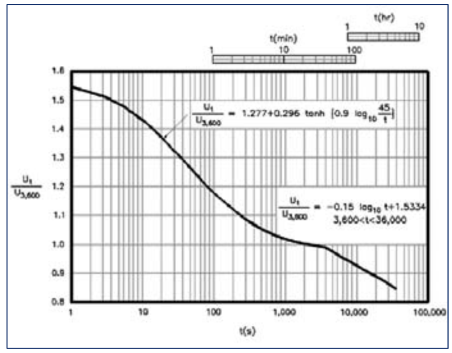
Wind speed adjustments

1. Adjust wind speed to equivalent speed at 10 m

$$U_{10} = U_z \left(\frac{10}{z}\right)^{1/7}$$

   - U_{10} = wind speed adjusted to 10 m
   - U_z = wind speed measured at height z (m)

2. Adjust wind speed to 1-hr average per graph
   - See CEM II-2 for more details



Wind speed adjustments (continued)

3. Adjust for air-sea temperature difference
   - $\Delta T = T_{air} - T_{water}$ (°C)
   - If variable (usual case), assume zero
   - See CEM II-2 for more details

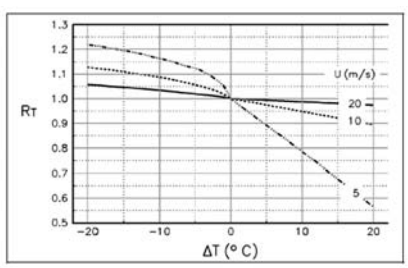


Figure II-2-8. Amplification R_T ratio of W_c (wind speed accounting for effects of air-sea temperature difference) to W_w (wind speed over water without temperature effects)

Wind speed adjustments (continued)

4. Adjust for effects of land drag

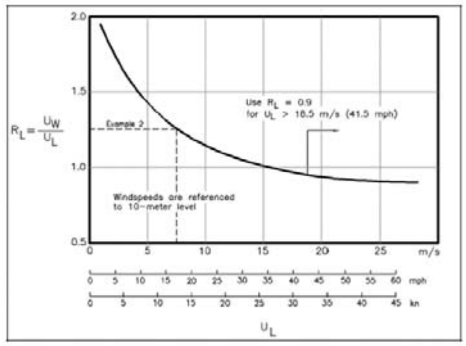


Figure II-2-7. Ratio R_L of windspeed over water U_w to windspeed over land U_L as a function of windspeed over land U_L (after Resio and Vincent (1977))

Fetch-limited sea state

- Is the sea state fetch-limited?

$$t_{X,u} = 77.23 \frac{X^{0.67}}{u^{0.34} g^{0.33}}$$

  - $t_{X,u}$ = time for waves over fetch X to become fetch-limited
  - If wind duration is at least $t_{X,u}$, the sea state is *fetch-limited*
  - If less, it's *duration-limited*
  – X = straight-line fetch (distance over which the wind blows)
  – H_{mo} = zero-moment wave height ≈ H_{sig}
  – u_* = friction velocity = $\sqrt{C_D} U_{10}$
  – C_D = drag coefficient
  – U_{10} = adjusted 10-m wind speed
  – T_p = period of peak wave energy

$$\frac{gH_{mo}}{{u_*}^2} = 4.13 \times 10^{-2} \left(\frac{gX}{{u_*}^2}\right)^{1/2}$$

$$\frac{gT_p}{u_*} = \frac{1}{2.727} \left(\frac{gX}{{u_*}^2}\right)^{1/3}$$

$$C_D = \frac{{u_*}^2}{{U_{10}}^2}$$

$$C_D = 0.001(1.1 + 0.035U_{10})$$

Duration-limited sea state

- $t < t_{x,u}$
- Find equivalent fetch, X_t, for *t (duration-limited)*

$$\frac{gX_t}{{u_*}^2} = 5.23 \times 10^{-3} \left(\frac{gt}{u_*}\right)^{3/2}$$

- *Use X_t in fetch-limited equations to estimate H_{mo} and T_p*

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Fully developed sea state

- No fetch or duration limitations
- Deep water

$$\frac{gH_{mo}}{{u_*}^2} = 2.115 \times 10^2$$

$$\frac{gT_p}{u_*} = 2.398 \times 10^2$$

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What is H_{mo}?

- A "moment" of a wave energy spectrum *E(f)*:
  - $m_n = \int_0^\infty E(f) f^n df$
  - *n = 0* for the "zero moment"
  - In general, the zero-moment wave height $H_{mo} = 4\sqrt{m_0} \cong H_s$
  - Quick estimate from water level data:
    - Since $m_0 \cong$ variance of a water level time series
    - $H_s \approx 4\sigma$
    - σ = standard deviation of water level time series, relative to mean

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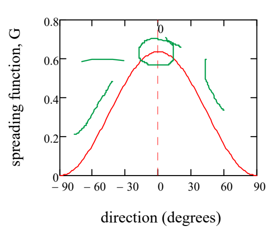
Directional spectra

- Wave energy spreads laterally outward from source
- Dominant direction prevails
- *S(f, θ)* = directional spectral density (m²/Hz/deg) = *S(f)G(f, θ)*
  - θ = direction of wave propagation (CCW is positive, right-hand rule)
  - *G(f, θ)* = directional spreading function (non-dimensional), relative magnitude of *S(f)* at each θ
    - *e.g.*, a ½-plane cosine distribution:

$$G(f,\theta) = G(\theta) = \frac{2}{\pi}\cos^2\theta \quad \text{for} \quad |\theta| \le \frac{\pi}{2}$$

$$G(\theta) = 0 \quad \text{for} \quad |\theta| \rangle \frac{\pi}{2}$$



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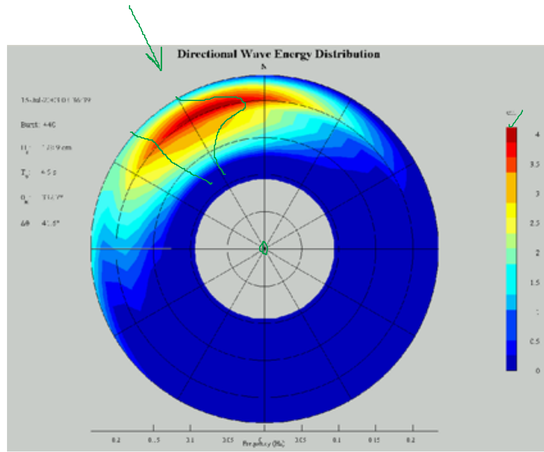
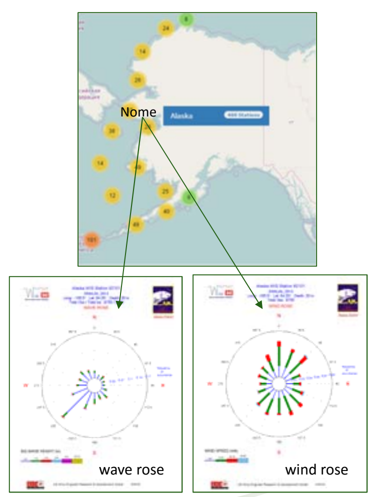
Measured directional spectra

- The energy density spectrum in any direction is associated with the color code
- Dominant direction is from NW



Hindcasting wave conditions

- Reconstruct surface winds from isobaric charts
- Model generation and propagation of waves
- Useful for compilation of wave climate statistics
- Useful for forensic investigations of disastrous storms
- Alaska wave hindcast database at http://wis.usace.army.mil/
  - 469 stations: western Alaska
  - Review WIS documentation for procedure background
  - Wind and wave statistics, plus individual event parameters



Extreme sea state statistics

- Use representative H_s; only extremes, not mild conditions
- Need multi-year record; the longer, the better
- Cumulative probability function, F(x)
  - Probability that x ≤ x' (a value of interest)
  - Extremal Type I (Gumbel): $F(x) = e^{-e^{\left(-\frac{(x-\mu)}{\sigma}\right)}}$
  - Log-normal: $F(x) = e^{-\frac{x^{-\mu}}{\sigma}}$
  - Weibull: $F(x) = 1.0 - e^{-\left(\frac{x-\mu}{\sigma}\right)^C}$

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Extremal analysis

1. Segregate extremes from record; *e.g.*, H_s > 2.5 m
   - λ = extremes per year; choose H_s for λ = 2-3
2. *n* values in ascending order; *k* is place in order (1, 2…*n*)
3. Estimate *F* by: $\hat{F}_k = \frac{k}{n+1}$
4. Transform Extremal Type I by: $-\ln(-\ln(\hat{F}(x))) = \frac{1}{\sigma}x + \frac{\mu}{\sigma}$
   - This form is the equation of a line: $y = ax + b$ (slope *a*, y-intercept *b*)
     - H_s values = *x*; $y = -\ln(-\ln(\hat{F}(x)))$

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Extremal Analysis (continued)

5. Compute function parameters from data: $F(x) = e^{-e^{\left(-\frac{(x-\mu)}{\sigma}\right)}}$
   - $\sigma = {}^{1}/_{slope}$ $\mu = \sigma \times intercept$
6. Return period (years): $T_{return} = \dfrac{1}{\lambda(1 - F(x))}$
7. Non-encounter probability: $NE(x) = e^{-\frac{L}{T_{return}}}$
   - probability for design life L, return period T_{return}, largest condition $\leq x$
   - For $L = T_{return}$, $NE(x) = 0.37$; i.e., 63% chance x will be exceeded

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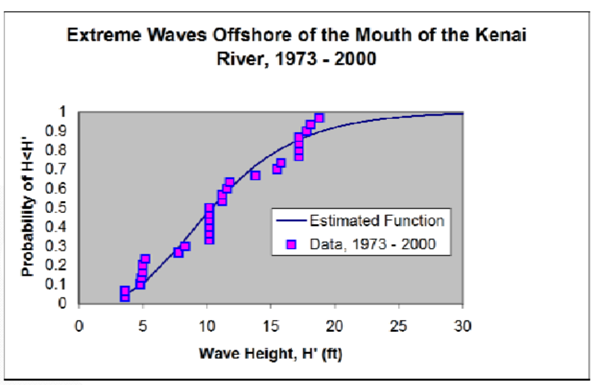
Extreme wave estimate, Kenai, AK

- Analysis by UAA grad student Heike Merkel
- sponsored by PND, Inc., and City of Kenai



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Conclusion
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