AN EMPIRICAL MODEL FOR SHIP-GENERATED WAVES

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Abstract: A new model for the prediction of ship-generated wave heights has been developed based on an empirical analysis of more then 1,200 data points from small-scale model tests reported in the literature. The new model was then validated at full-scale in field trials conducted using training vessel owned by the U.S. Naval Academy. Predictions using the new model appear to be superior to those made using the more established model of Sorensen and Weggel.

INTRODUCTION

This paper presents a new empirical model for the prediction of maximum ship-generated wave heights. The model was developed using a large database of ship-generated waves obtained from more than 16 sources in the literature, mostly based on laboratory tests. This data was then augmented by new data obtained from towing tank tests carried out at the U.S. Naval Academy. In addition, a series of full-scale field trials were conducted and these were used as a blind test of the predictive capability of the new empirical model. While much recent interest in ship waves has focused on the wake wash from high speed ferries, the emphasis in this work was on more traditional displacement vessels operating at sub-critical Froude numbers.

BACKGROUND

In recent work for the U.S. Navy, Seelig and Kriebel (2001) consolidated published data on ship-generated waves from original reference sources into a common database. As of this writing, this database includes results for 60 individual vessels obtained from 16 papers and reports published in the literature. Almost all of this data is from small scale model tests, and most data are from model tests conducted in the 1960's. One limitation on the database is the limited amount of full-scale field data on ship-generated wave heights for conventional displacement vessels.

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Figure 1 shows a definition sketch for ship-generated waves caused by a moving vessel. Each entry in the database consists of the maximum wave height measured at a distance *y* off the sailing line, for a given ship (with length, *L*, draft, *D*, and other hull form characteristics), moving at a specific speed, *V*, in water depth, *d*. For each ship model, tests were typically conducted for several water depths so that wave measurements are available for several values of the relative draft, *D/d*. For each of these relative depth conditions, several (usually 5 to 10) ship speeds were tested and wave measurements were obtained for several (usually 3 to 10) distances from the sailing line.

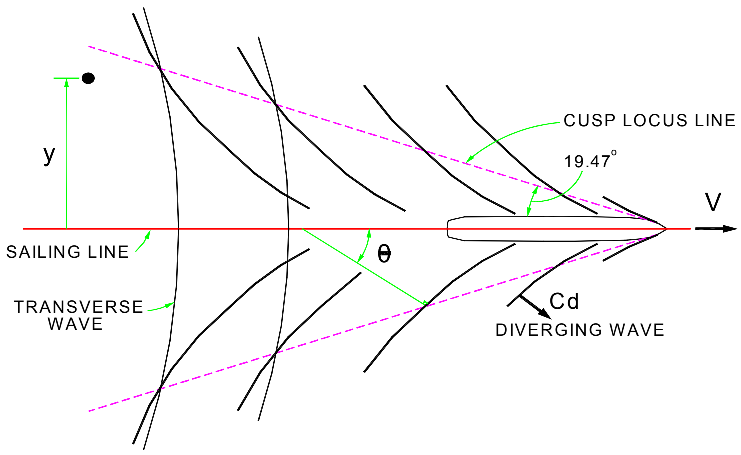

Figure 1. Definition sketch showing plan-form of ship-generated waves

In the present analysis, a subset of the data was selected for use in developing an empirical model for the maximum ship-generated wave height. Original data was obtained from papers and reports by Biddie (1968), Carruthers (1966), Das (1969), Hay (1968), Helwig (1966), Kurata and Oda (1984), and Sorensen (1966). In order to augment the published data, additional data on ship-generated waves have been collected in a series of physical models tests conducted at the U.S. Naval Academy. Tests have been conducted in deep water in a large towing tank, and in shallow water using a smaller towing tank. The main emphasis in these tests has been to fill in data gaps identified from the literature search. Model tests used a Series 60 ship model, a hull form that has been widely used in towing tanks around the world and one that is representative of a generic cargo ship. Altogether, the data set used to develop the empirical model for ship-generated waves included more than 2,100 data points for 12 ship models.

The main goal of this study is to improve upon the empirical model for ship waves developed by Sorensen and Weggel (1984) and Weggel and Sorensen (1986). In the Sorensen and Weggel (S&W) model, all length scales are normalized by the cube-root of the ship's displaced volume, and the dimensionless wave height is then given as a function of dimensionless distance from the sailing line and the dimensionless water depth, through use of five empirical parameters. Each of these five empirical parameters are functions of the depth-based Froude number, $Fd=V/(gd)^{1/2}$.

While the S&W model has been widely used, it also has some shortcomings. First, none of the empirical parameters in the model are functions of the ship hull shape and results are dependent only on the cube-root of the displaced volume, and not on the hull form. Second, it is difficult to gain physical insight from the model. For example, the dependence on ship speed is contained in each of the five empirical parameters, so that the functional relationship between H and V is masked. Similarly, the decay of wave heights with distance off the sailing line is a complicated function of two empirical parameters and is difficult to discern. Finally, blind validation testing of the S&W model using field data appears to be lacking.

NEW EMPIRICAL MODEL

To develop the new empirical model, dimensional values from the many model tests were first normalized. Based on physical arguments, the amplitude of the bow wave for a moving ship should scale according to the velocity head, $V^2/2g$, suggesting that wave heights can be normalized in the form gH/V^2. An advantage of this form is that normalized wave heights should never exceed unity, thus placing a rational upper bound on the wave height data. The location relative to the sailing line was then normalized by the ship length as y/L. In early stages of the project, other horizontal length scales were evaluated, i.e. ship beam, but ship length worked as well as any and was retained because it has been widely used in the past.

The next phase of the study involved a determination of a functional form for wave attenuation with distance from the sailing line. In the literature, it appears that two simple models have been suggested: $(y/L)^{-1/3}$ and $(y/L)^{-1/2}$. From theoretical arguments, the -1/3 model appears to apply to the diverging waves while the -1/2 model appears better for the transverse waves. Preliminary data analysis involved plotting normalized wave heights versus the normalized distance from the sailing line, and fitting both models to determine which gave the best fit. From this analysis, it was determined that -1/3 model provided the best fit of the majority of the 1,200 data points analyzed and this was adopted for further use.

Figure 2 shows some examples of the $(y/L)^{-1/3}$ model fit to data, two of which show good agreement with the -1/3 model and one of which would be better fit with and exponent of -1 or so. It is noted that best-fit exponents were also determined for each set of tests in the database, and that these values ranged from about -0.2 to -1.5. While no simple trend was evident, it was evident that exponents of about -0.333 seemed to work best for the higher ship speeds tested. As a result, the exponential decay adopted here seems representative for conditions of the most interest for generation of the largest ship waves.

Once the $(y/L)^{-1/3}$ model was adopted, the database was further simplified by summarizing each set of tests by the value of gH/V^2 at $y/L=1$, illustrated by the bold circles in Figure 2. In this way, all tests conducted for a given ship, with a given depth-to-draft ratio, and moving at a given speed could be characterized by a single representative wave height isolated from the effects of distance from the sailing line.

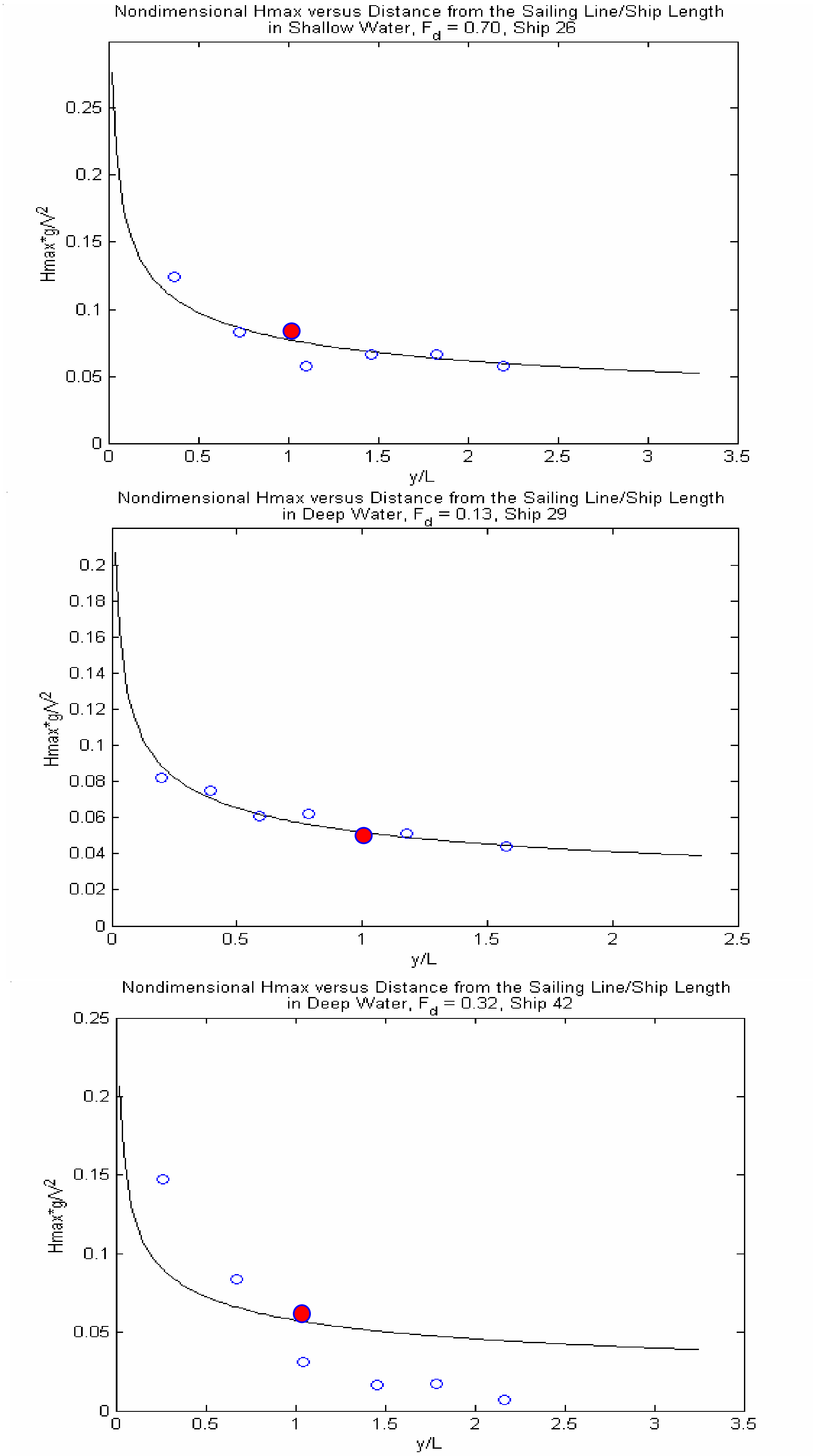

Figure 2. Examples of wave height decay with distance from sailing line.

Model development then focused on how the characteristic values of gH/V^2 at $y/L=1$, for a given ship with a given depth-to-draft ratio, varied as a function of ship speed. Because of the desire to develop a model that could be applied across a range of water depths, the means by which the ship speed was normalized was problematic. In deep water, it is well known that ship-generated waves vary with a length-based Froude number, $F_L=V/(gL)^{1/2}$. In contrast, it is also well-known that in shallow water, ship-waves are better described by a depth-based Froude number, $F_d=V/(gd)^{1/2}$. Observations indicate that the length-based Froude number is most appropriate when the depth-to-draft ratio, d/D, is greater than about 5 or so, while the depth-based Froude number is most appropriate when the depth-to-draft ratio is less than 1.5. For situations in between, which constituted much of the database, neither the length-based nor the depth-based Froude numbers seems fully appropriate.

After working with the data, we discovered that an empirically modified Froude number seemed to provide a better and more unified description of ship-generated waves than either F_L or F_d.. This modified Froude number, F^*, has the following form

$$F_* = F_L \exp(\alpha D/d) = \frac{V}{\sqrt{gL}} \exp(\alpha D/d) \tag{1}$$

where the parameter α was selected so that, when values of gH/V^2 at $y/L=1$ were plotted against this modified Froude number, data for a given ship from all speeds and water depths seem to collapse to a nearly single curve with a unique value of the coefficient α. Furthermore, the data from various ships showed some common behavior. For values of F^* below 0.1, wave heights were negligible as ship speeds were too low to generate any measurable waves in the small-scale lab tests. As F^* increased, the values of gH/V^2 at $y/L=1$ then increased. While the functional form of this increase varied somewhat for each hull form, a quadratic increase was found to provide a good overall description of the measurements.

Figure 3 shows examples of how the values of gH/V^2 at $y/L=1$ varied with F^* for various ship models. The data shown are for various values of water depth and ship draft (each value of D/d plotted using a unique color and symbol), and it is apparent that results from tests conducted at various values of d/D seem to collapse to a single curve for each ship model. The smooth curve shown in each figure is a quadratic function for values of F^* greater than 0.1, that has been fit through the data.

As a result of this analysis, a new empirical model for ship-generated wave heights was developed having the following form:

$$\frac{gH}{V^2} = \beta\left(F^* - 0.1\right)^2 \left(\frac{y}{L}\right)^{-1/3} \tag{2}$$

where β is an empirical coefficient that depends on hull form. As shown in equation (1), the modified Froude number, F^*, contains an empirical coefficient, α, that also depends on hull form.

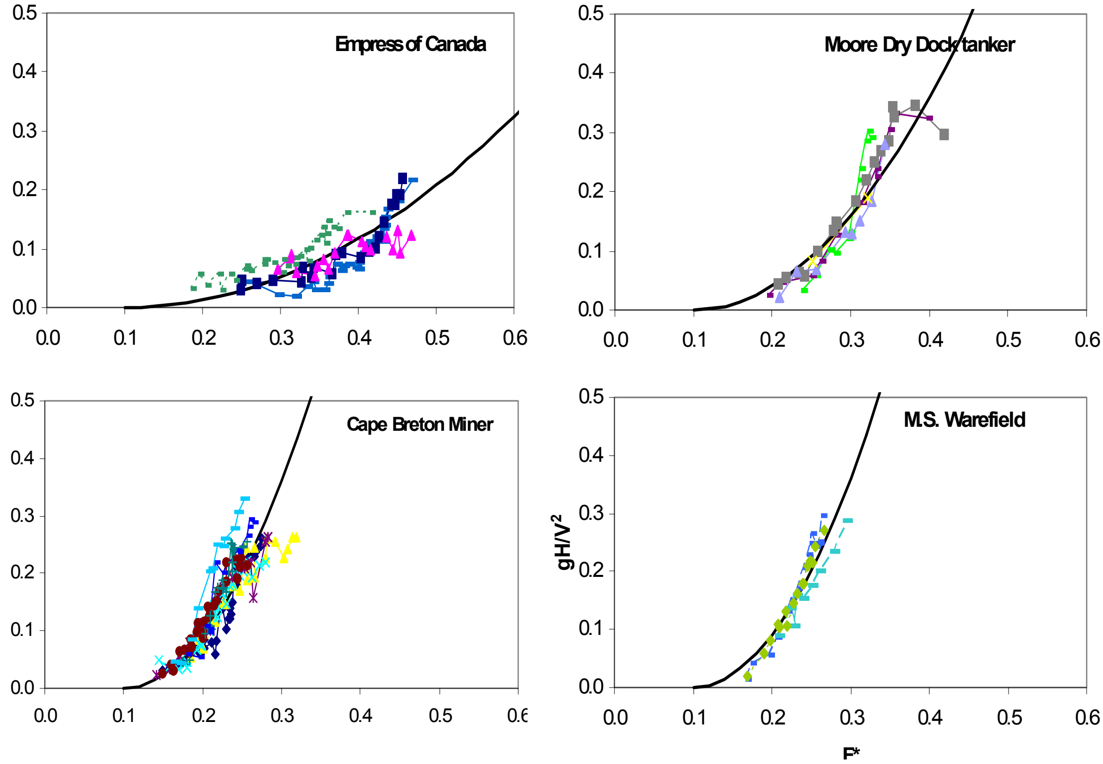

Figure 3. Examples of gH/V^2 at $y/L=1$ plotted versus modified Froude number F^*. For each ship, the use of F^* seems to collapse results from many depth-to-draft ratios (different symbols/colors) to a single curve. Also shown are best-fit quadratic curves for $F^*>0.1$.

We have then investigated the two empirical parameters, α and β, and have found that both varied systematically with shape parameters related to the hull form. As shown in Figure 4a, the coefficient α used to define F^* was found to depend on the ship block coefficient in the form:

$$\alpha = 2.35(1 - C_b) \qquad (3)$$

where

$$C_b = \nabla / (L\,B\,D)$$

The parameter β was more problematic but appeared to depend on the shape of the bow. Borrowing an idea from Gates and Herbich (1975), we eventually found that β varied with the bow entry length, *Le*, which is defined as the distance from the bow to the widest part of the hull. For slender streamlined hulls, *Le* can be as much as nearly one-half the ship length and this lessens the wave-generating potential. On the other hand, blunt bows with a small entry length have larger wave generating potential. As shown in Figure 4b, these trends were reflected in the best-fit values of b, and an empirical fit of the dependence was determined as

$$\beta = 1 + 8 * \tanh^3\left(0.45\left(\frac{L}{L_e} - 2\right)\right) \qquad (4)$$

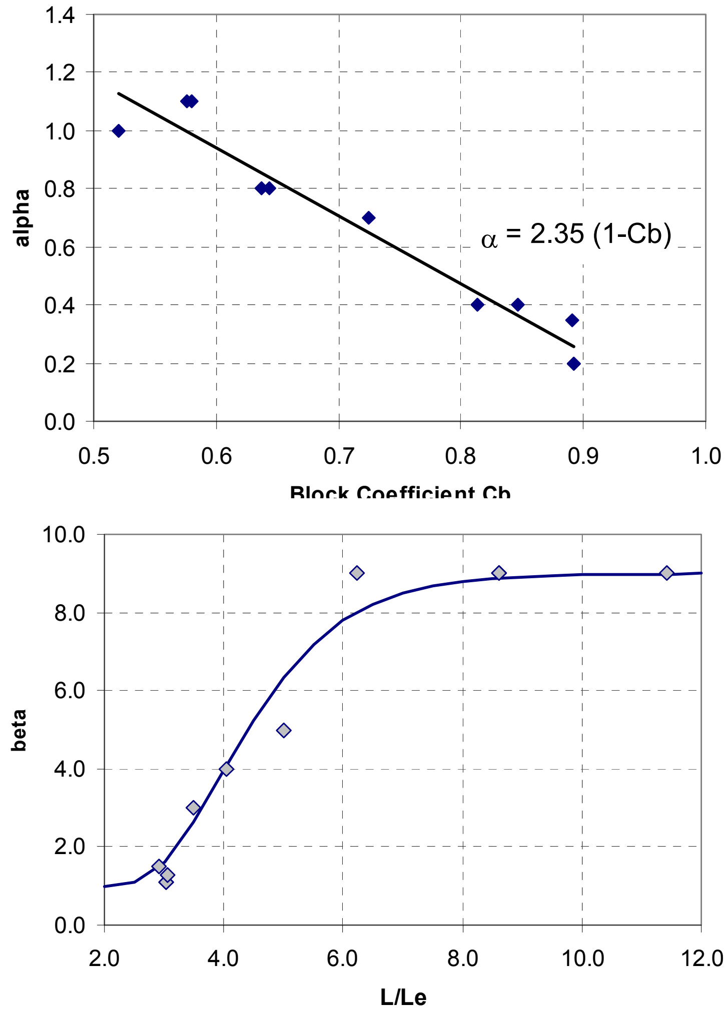

Figure 4. Top figure (a) showing parameter α used to define the modified Froude number. Bottom figure (b) showing the parameter β used to define the rate of increase of gH/V^2 with the modified Froude number.

FIELD TRIALS

In order to evaluate both the Sorensen and Weggel (S&W) model and the new empirical model developed in equations (1) to (4), a set of field trials was conducted to measure ship generated waves. Using this data, both models were applied in a blind validation test using empirical parameters as determined in model development without further refinement.

The field trials were conducted in the Chesapeake Bay near Annapolis, Maryland. Tests were conducted under near ideal conditions of little to no wind. The water surface started glassy and had wind waves up to a few centimeters by the end of the testing. Water depths across the study site vary from about 5m to 7 m. A single wave gage was used in the study, located in a depth of 5.5m. In areas traversed by the ship, depths were a little deeper and averaged about 6m.

The vessel used in the trials is a small naval training vessel owned by the Naval Academy, the YP686, shown in Figure 5. This vessel has a waterline length L=31.1m, a maximum beam B=6.5m, and a draft D=1.83m. The displaced volume, ∇ = 154.7 m^3, the block coefficient, C_B = 0.41, and the ratio of entry length to ship length, Le/L = 0.4. Four ship speeds were selected, V = 3.6, 4.1, 4.6, and 5.1 m/sec (7, 8, 9, and 10 knots). For each speed, trials were conducted with the ship passing at distances of y = 15, 30, 61, 91, and 122 meters (50, 100, 200, 300, and 400 feet). These distances relative to the wave gage location are not highly accurate and were determined using a GPS chart plotter on board the vessel.

Figure 5. Vessel used in field trials, the YP686. Vessel is shown passing wave gage. The wave staff is visible just aft of midships.

For each trial, the maximum ship-generated wave heights were determined and then normalized by the velocity head. Results are plotted in Figure 6 as a function of distance from the sailing line. Also plotted are predictions obtained using the new empirical model. Figure 7 shows the same data along with predictions using the Sorensen and Weggel (S&W) model. It is noted that the S&W model has been refined by Weggel and Sorensen (1986) to include corrections to the original model based on ship hull form. These corrections are tabulated for the vessels used to derive their model, but cannot be determined for a new vessel not included in their database. As a result, their results are given here in the uncorrected form.

Results generally indicate that the new empirical model provide improved predictions of the full-scale ship generated wave heights for the conditions tested. Results of the new model are quite good for the three highest speeds tested of 8, 9, and 10 knots. The magnitude of the ship-generated waves is well-predicted at y/L=1, the location used to develop the empirical relationships, and the attenuation with distance from the sailing line is well-predicted using the $(y/L)^{-1/3}$ function. It is noted, however, that there is considerable scatter in some of the data, as evidenced by the data for the 9 knot trials which were run twice. The model tends to underpredict for the lowest speed of 7 knots. The data for this case appears problematic, however, as the wave heights were found to increase with increasing distance from the sailing line. This is due in part to the small wave heights observed for this speed, and the difficulty in separating the ship waves from the background wind waves.

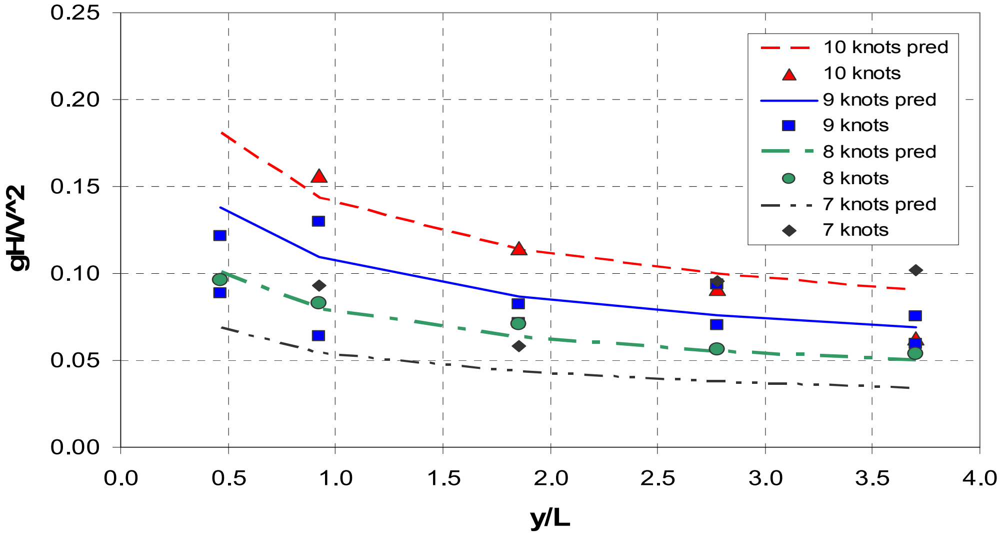

Figure 6 Results of the field trials along with prediction using new empirical model in equations (1) through (4).

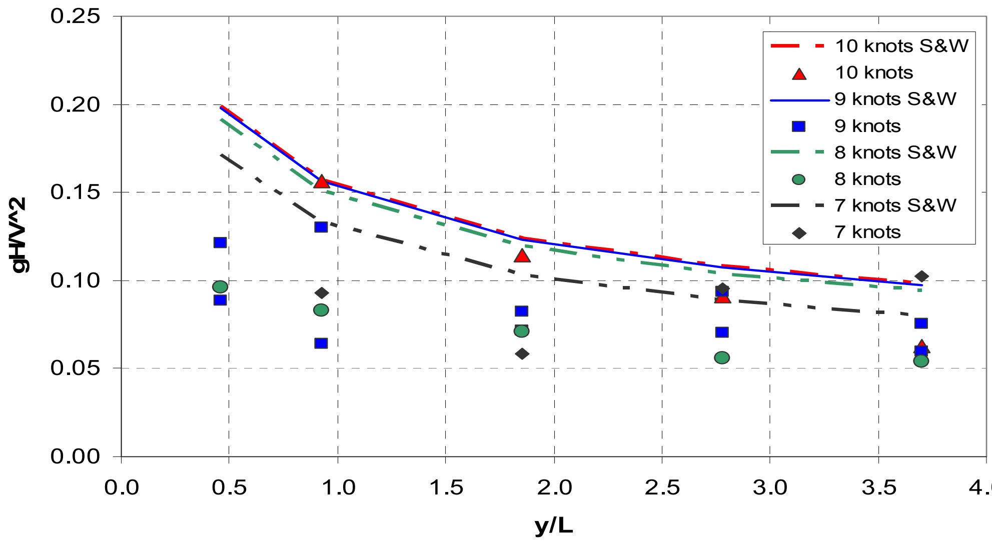

Figure 7. Results of the field trials along with prediction using Sorensen and Weggel model

The S&W model, in Figure 7, was found to overpredict the ship generated waves in all but the highest speed tested. The S&W predictions also did not vary as strongly with ship speed as the data suggests. Predictions for 8, 9, and 10 knots were almost identical, while the data showed a clear increase in normalized wave height with increasing ship speed. The near uniformity in the predicted wave heights for various speeds suggests that the S&W model gives a functional relationship in which wave heights are generally proportional to V^2. The new empirical model contains a much stronger relationship with wave height proportional to V^4 to leading order.

CONCLUSIONS

This paper has summarized the development of a new empirical model for predicting ship-generated wave heights, and it has included preliminary evaluation of the model using field data from full-scale ship trials. The model, given in equation (2) with parameters defined in equations (1), (3), and (4), appears to provide an improved prediction of ship-generated waves when compared to the well-established model of Sorensen and Weggel. The key element of this new model is the definition of a modified Froude number, F*, that incorporate ship speed and the relative depth-to-draft ratio. For any given ship, this parameter seems to collapse wave height data from many different depths and ship drafts into a single curve. Until the new model is evaluated more rigorously, it should only be applied within the limits of the data used in its development. It is suggested that the model should be applied for values of F^* in the range $0.1<F^*<0.5$. In addition, when normalized wave heights at $y/L=1$ are considered, no data in the database exceeded $gH/V^2=0.4$. As a result, the model should be applied only when the quantity $\beta(F^*-0.1)^2<0.4$.

ACKNOWLEDGEMENTS

The authors would like to thank Dr. Carolyn Judge, who evaluated the attenuation of ship waves with distance from the sailing line, Ms. Chris Fleurant, who coordinated use of the YP686, and Midshipman Seth Saalfeld, who worked on the field data collection as part of a student project.

REFERENCES

Biddie, D.D., 1968, “Ship Waves in Shoaling Water,”, Technical Report HEL 12-6, Univ of California.
Carruthers, J.G., 1966, “Ship Waves and Drawdown”, Queens University thesis.
Das, M.M., 1969, “Relative Effect of Waves Generated by Large Ships and Small boats in Restricted Waterways”, Report HEL-12-9, Univ of California.
Hay, D., 1967, “Ship Waves in Navigable Waterways”, Report HEL 12-5, Univ of California.
Helwig, P.C., 1966, “An experimental study of ship generated water waves”, Queens University thesis.
Kurata, K. and Oda, K., 1984, “Ship Waves in Shallow Water and Their Effects on Moored Small Vessel”, Proc Coastal Engineering Conference, pp. 3258-3273.
Seelig, W. N. and Kriebel, D.L., 2001, “Ship-Generated Waves,” Naval Facilities Engineering Service Center, Tech.Rpt. TR-6022-OCN
Sorensen, R.M., 1966, “Ship Waves,”, Technical Report HEL 12-2, University of California.
Sorensen, R. M. and Weggel, J.R., 1984, “Development of Ship Wave Design Information”, Proc Coastal Engineering Conference, pp. 3227-3243.
Weggel, J. and Sorensen, R.,1986, “Ship Wave Prediction for Port and Channel Design”, ASCE Ports ’86, pp. 797-814.